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Topology and its Applications 153 (2006) 1100–1102
Topology and its Applications
www.elsevier.com/locate/topol

Corrigendum

Corrigendum to: "Orbit spaces and unions of equivariant absolute neighborhood extensors" [Topology Appl. 146–147 (2005) 289–315]

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Received 29 January 2005; received in revised form 22 February 2005; accepted 23 February 2005

MSC: 54C55; 54C20; 54H15

Keywords: *G-ANR*; Equivariant homotopy; *H*-orbit space

In our article [3], on p. 313, the proof of Theorem 6.1 contains a gap. Namely, in that proof we concluded that the orbit space X/H is a G/H-*ANR*, by applying the equivariant version of the Hanner domination theorem [2, Theorem 7]. The error occurs at the place where we verify only the weaker condition that the two equivariant maps in question are β-close, β being an open cover, while the assumption of the equivariant Hanner domination theorem requires these two maps to be equivariantly β-homotopic.

Using the notation adopted in [3], this gap may be filled as follows.

Let X be a topological space and γ an open cover of X. Two continuous maps $f, \varphi: Y \to X$ are called γ-close, if for every $y \in Y$ there exists $\Gamma \in \gamma$ such that $\{f(y), \varphi(y)\} \subset \Gamma$.

DOI of original article: 10.1016/j.topol.2003.05.004.
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[1] The author was supported in part by grant IN105803-3 from PAPIIT (UNAM).

0166-8641/$ – see front matter
doi:10.1016/j.topol.2005.02.009

A homotopy $F_t : Y \to X$, $t \in [0,1]$, is said to be limited by γ, or simply, a γ-homotopy provided for any $y \in Y$, there exists $\Gamma \in \gamma$ such that $F_t(y) \in \Gamma$ for all $t \in [0,1]$. If, in addition, X and Y are G-spaces then a γ-homotopy (F_t) is called equivariant or a G-homotopy, if F_t is a G-map for every $t \in [0,1]$. In such a case F_0 and F_1 are called equivariantly γ-homotopic G-maps.

We shall say that a G-space X is equivariantly γ-dominated by a G-space Y, if there exist G-maps $f : X \to Y$ and $\varphi : Y \to X$ such that φf is equivariantly γ-homotopic to the identity map 1_X of X.

Theorem 6.1. *Let G be a compact group, H a closed normal subgroup of G, and X a G-ANR* (*respectively, a G-AR*). *Then the H-orbit space X/H is a G/H-ANR* (*respectively, a G/H-AR*). *In particular, the G-orbit space X/G is an ANR* (*respectively, an AR*).

Proof (*new*). We are going to apply the following equivariant Hanner domination theorem, established in [2, Theorem 7]: if for any open cover β of a metrizable G-space M, there exists a G-*ANE* that equivariantly β-dominates M, then M is a G-*ANR*.

Now let β be an open cover of the metrizable G/H-space X/H, and let $\alpha = p^{-1}(\beta) = \{p^{-1}(V) \mid V \in \beta\}$, where $p : X \to X/H$ is the H-orbit projection.

Since X is a G-*ANR*, by [1, Proposition 3], there exists an open refinement γ of α such that any two γ-close G-maps $f_0, f_1 : Y \to X$ are equivariantly α-homotopic.

Then, by Theorem 4.6 of [3], there exist G-maps

$$f : X \to \mathcal{N}(\mathcal{U}) \quad \text{and} \quad \psi : \mathcal{N}(\mathcal{U}) \to X$$

such that the composition ψf is γ-close to the identity map of X, where $\mathcal{N}(\mathcal{U})$ is the G-nerve of a G-normal cover $\mathcal{U}$ of X. Consequently, ψf is equivariantly α-homotopic to the identity map of X. Let $F_t : X \to X$, $t \in [0,1]$, be a G-homotopy limited by α such that $F_0 = \mathrm{Id}_X$ and $F_1 = \psi f$.

Passing to the H-orbit spaces, we get continuous G/H-equivariant maps

$$\tilde{f} : X/H \to \mathcal{N}(\mathcal{U})/H \quad \text{and} \quad \tilde{\psi} : \mathcal{N}(\mathcal{U})/H \to X/H,$$

and a G/H-homotopy $\widetilde{F}_t : X/H \to X/H$, $t \in [0,1]$, such that $\widetilde{F}_0 = Id_{X/H}$ and $\widetilde{F}_1 = \tilde{\psi}\tilde{f}$.

Let us check that the induced G/H-homotopy $(\widetilde{F}_t)$ is limited by the cover β. Indeed, let $p(x) \in X/H$ be arbitrary, where $x \in X$. Since (F_t) is limited by α, there exists an element $W \in \alpha$ such that $F_t(x) \in W$ for all $t \in [0,1]$. By definition of α, there exists $V \in \beta$ such that $W = p^{-1}(V)$. Then $\widetilde{F}_t(p(x)) = p(F_t(x)) \in p(W) \subset V$ for all $t \in [0,1]$, as required.

Thus, X/H is G/H-equivariantly β-dominated by $N(\mathcal{U})/H$. Since by Lemma 6.2, $N(\mathcal{U})/H$ is a G/H-*ANE*, it follows from the above mentioned equivariant Hanner domination theorem that X/H is a G/H-*ANR*.

If $X \in G$-*AR*, then $X \in G$-*ANR* and X is G-contractible. This implies easily that X/H is G/H-contractible. Since by the preceding case X/H is a G/H-*ANR*, we conclude that X/H is a G/H-*AR*. This completes the proof. □

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