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Note

A counterexample to a conjecture on edge-coloured tournaments

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Abstract

We call the tournament T an m-coloured tournament if the arcs of T are coloured with m colours. In this paper we prove that for each $n \geqslant 6$, there exists a 4-coloured tournament T_n of order n satisfying the two following conditions: (1) T_n does not contain C_3 (the directed cycle of length 3, whose arcs are coloured with three distinct colours), and (2) T_n does not contain any vertex v such that for every other vertex x of T_n, there is a monochromatic directed path from x to v. This answers a question proposed by Shen Minggang in 1988.

MSC: 05C20

Keywords: Edge coloured tournament; Monochromatic directed path

A directed path is called monochromatic if all of its arcs are coloured alike.

In [2], Sands, Sauer and Woodrow proved that every 2-coloured tournament T has a vertex v such that for every other vertex x of T there is a monochromatic directed path from x to v. They also raised the following problem: Let T be a 3-coloured tournament which does not contain C_3. Must T contain a vertex v such that for every other vertex x of T there is a monochromatic directed path from x to v?

In [1] Shen Minggang proved that if T is an m-coloured tournament which does not contain C_3 or T_3 (the transitive tournament of order 3, whose arcs are coloured with three distinct colours). Then there is a vertex v of T such that for every other vertex x of T there is a monochromatic directed path from x to v. He also proved that the situation is best possible for $m \geqslant 5$. The question for $m=3$ (the problem raised by Sands et al.) and the respective question for $m=4$ (if T is a 4-coloured tournament which does not contain C_3, then T has a vertex v such that for any other vertex x of T, there exists an xv-monochromatic directed path) were still open.

In this paper we construct a family of counterexamples to the question for $m=4$. The question for $m=3$ is still open.

Theorem 1. *For each $n \geqslant 6$ there exists a 4-coloured tournament of order n satisfying the two following conditions*: (1) *T_n does not contain C_3, and* (2) *T_n does not contain any vertex v such that for every other vertex x of T_n, there is a monochromatic directed path from x to v.*

Proof. For $n=6$, the tournament T_6 in Fig. 1 is a 4-coloured tournament, of order 6, contains no 3-coloured directed cycle of length 3. And T_6 does not contain any vertex v such that for any other vertex x of T_6 there is a monochromatic

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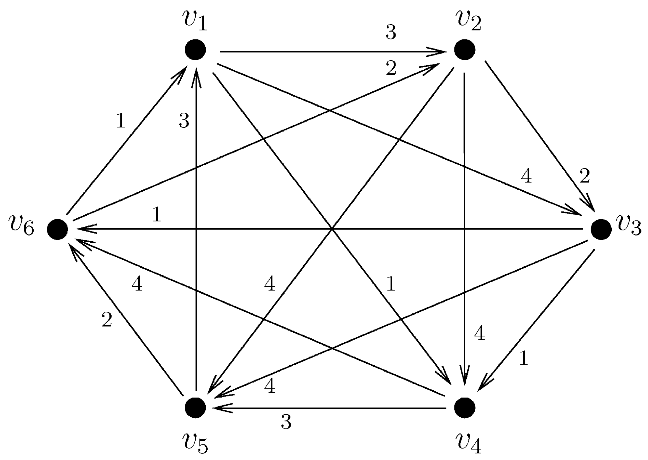

Fig. 1.

directed path from x to v. In fact v_{i+1} cannot reach v_i via a monochromatic directed path, where the notation is taken module 6.

Larger 4-coloured tournaments with the same properties of T_6 can be constructed by adding vertices to T_6, one at a time, connecting each new vertex to all previous vertices by an arc coloured 1. □

References

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